FUNdamental Mathematics

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A Voyage into the Quirky Universe of Maths and Jokes

David Eelbode

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Foreword

I have no particular talent,
I am only inquisitive.
(Albert Einstein[1])

At some point during our lives, we all feel genuinely misunderstood. A common fact, almost as universal as the law which says that when you wake up in the middle of the night, wondering whether it's safe to ignore your pressing bladder until the next morning, the answer is always and invariably 'no'. If not during puberty — that time when we all feel misunderstood by our parents and, give or take, the rest of the universe — then at least during excessive Friday night pub crawls, when an unfortunate audience is forced to take our self-proclaimed Words of Genius for what they really are: bags of heavily intoxicated mental rubbish, lining the shadier corridors of our brain like abandoned couches and broken televisions, joylessly waiting to be picked up during the weekly garbage truck procession.

Being a mathematician, feeling misunderstood is part of my repertoire, together with making people at parties feel rather uncomfortable when they ask me what I do for a living, and knowing how much I will have to pay for my groceries, even before the cashier starts sliding my stuff across the miniature party laser light system, marking the end of the conveyor belt.

This is one of the reasons why I decided to write the book you are now holding in your hands: by the time you have reached its final sentences, I hope that you will have gained some insight into the workings of a mathematically inclined mind. Because it may not always be that obvious, but whenever it is not concentrating on a

1 Every text needs at least one Einstein quote.

calculation or a proof, nor longing for another cup of coffee, the somewhat overdeveloped mental muscle sitting on top of my body is essentially trying to do what yours is doing: getting a firmer grip on this thing called 'daily life'. In a rather peculiar way though, since it is only fair to say that my bulimic brain has a ravenous appetite for food for thought. Like a baseball coach in his dugout, my brain is ferociously munching its way through the days, masticating even the most mundane of facts, processing them into propositions which can be tested and perfected, trying to translate the results into theorems that can survive the times. Or at least your attention span, as this book contains the fruits of my mental foraging over the last few years: definitions and theorems about things like tofu and polar bears, the mathematician's way of trying to make sense of things.

The second reason why I decided to write this book stems from the fact that I never really understood why the vast majority of people consider my field of expertise, by and large, to be their least favourite subject at school. French and PE can basically do what they want to win this notorious race: at the end of the road, the Queen of the Sciences[2] will still be waiting for them. Freshly showered, casually solving Sudokus while enjoying a colourful cocktail. Deep down this makes me feel inherently sad — like seeing elderly men shuffle through the red light district on Christmas Eve. Or worse still, some people even try to upgrade their cool by admitting that they suck at mathematics — often blaming their high school teachers while they are at it. Something tells me that you cannot accomplish the same result when outing yourself as an illiterate person, right? With this book, therefore, I hope to build a neural connection between what may very well be two completely disconnected regions in your brain: the centrally located Skyscrapers of Fun, and the abandoned Maths Barracks.

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2 Not my words: it was Carl Friedrich Gauss (1777-1855) who once said that mathematics is the Queen of the Sciences.

In order to situate my third reason to write this book, we need to go back to 2015 — my very own annus horribilis. A year marked by burn-out and depression, two hands around the same throat. After an intense struggle at home — clinging to a sofa and a bottle of pills — I decided to leave Antwerp behind for a few months. It was either Antwerp or my life, so I had nothing to lose. I essentially felt like a reptile at the local zoo: everyone just assumed that I was still alive, because the light in my terrarium was still burning, but what people were really seeing was the faint afterglow of a comet that had plummeted through the atmosphere, crashing into rock bottom, producing a huge crater in which what once was a sparkly body of energy slowly started to decay into a pile of smouldering ashes and dust.

Until I finally found the courage to pull the plug out of my maddening mental fuse box: I decided to take unpaid leave, and went to Tokyo for four months. A surprising choice according to my friends and family, but it made perfect sense to me: Japan is an awe-inspiring universe in itself, with a giant attractor at its center. A pulsating heart which made my blood flow again too. It was in Tokyo — with its slender women who are often as wobbly on their high heels as the skyscrapers during one of the many earthquakes — that I regained my stability. It was there — in the world's capital of sleep deprivation, with its hard-working inhabitants who can doze off more easily on a packed train than I can fall asleep after three sleeping pills, a bottle of wine and a five hour rendition of a melting snowball — that I found the will to get up in the morning again. It was in Japan — where the contrasts are as sharp as the knives with which they fillet their tunas — that I shed my old skin. It was there, miles away from mathematics, that I realised how much I missed her. In a sense, this book thus doubles as a gigantic love letter: the music of reason was my first love, and it will be my last.

Over the past few months, people have often asked me: 'So what kind of book are you writing?' Right from the start I knew that I wanted to write the type of book that ends up on a coffee table.

Or on that little shelf in the toilet where most people keep a few outdated magazines and a crossword book. It can definitely be devoured in one go, but I actually recommend you to read a few pages and then leave it there for a while; like most of Tool's albums, olives and facial hair, it has to grow on you.

So is it a mathematical book? Yes and no. It most certainly looks like one — it contains definitions, theorems and formulas — but then again, it is not. I have merely attempted to combine my biggest passions in life (science, language, humour, food, travelling and music) throwing them into a blender, hoping to create something which can both make you laugh and teach you something. About maths, science and the venturings of an inquisitive mathematician in this world.

This book would have stayed an abstract concept forever — just like black hole tourism and me wearing my Isis t-shirts in public again[3] — if it wasn't for a few people who helped me turning it into the actual thing you are now holding in your hands. First of all, I would like to thank Lies (for going along with this crazy plan of mine in the first place), Isaac (for your valuable advice throughout the publication process) and Flore (who must have cursed me on more than one occasion for my constant nagging about ill-placed spacing and ugly fractions). Also Tamsin and Wouter deserve a special mention here: the latter is a good reader of proofs, the former is an outstanding proofreader.

Next, I would like to thank a few people who make my life on Planet Maths even better than it already was with maths only: Alberto (my one and only Klimentska-brother), Paul and Rudi (as prime as colleagues can possibly be), Annick and her posse, Mariska and Suzy, Greet and Christine (Eager Scientists for the win), Werner and Stijn, Bart (I promise not to mention

3 Isis used to be this utterly brilliant metal band, before the name was hijacked by malevolent people.

'Gobelijn' here), Matthias plus the Twin-Tim (if you ever start a band together, promise me to call it 'the Higher Spin Sons'), the Cliffordians at Ghent University (count Vladimir included) and my students (giving proper meaning to my life as a mathematician).

Finally, a big shout out to the Earthlings who are always eagerly awaiting me when I return from yet another trip in that vast space commonly referred to as 'my mind'. My parents (for allowing me to study maths in the first place), *watashi no nihonjin no kazoku* (see you at Ware), Wibbie and Macky (let that be a lesson, thou shalt not contract names), Nick and Ilse (owners of the only chihuahua that will ever matter), Elke and Sarah (VdM = Very dedicated Mates), HGP and the BLB-crew (I bet half of this book was conceived at your place), Evy (curiosity may have killed the cat, it did give birth to something better), Sofie and Tuur (I do hope you realise I will start reading this book out loud when you ask me to babysit), Laurent and Mathieu (<3) and Popol (luring me into the lovely arms of the Queen of the Sciences).

And you, of course, for buying (or any other verb, for that matter) this book and giving it a chance. Have fun along the way!

David Eelbode

Warning: this book contains functional nudity, an absolutely enormously huge amount of exaggerations, n mathematical symbols (with $n \in \mathbb{Z}^+$), fucking profanities, mathematical puns, footnotes, traces of peanuts,[4] dead puppies, a completely irrelevant amount of tofu, definitions (these are statements that explain the meaning of a word or phrase), ambiguous alliterations, mental mathematics, plenty of coffee, jokes which some scientists may find a little offensive,[5] small additions for math geeks only (where 'small' can be any $\epsilon > 0$), a few wombats and the absolute minimum of formulas.[6]

4 The sum over all peanuts on the diagonal.

5 I did try to make them easy enough, so that also the engineers can have a laugh.

6 The theoretical proof for this was eliminated by the editor though, as it involved an argument containing a lot of mathematical equations.

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1

A Space Odyssey

Space. It seems to go on and on forever.
Then you get to the end, and a monkey
starts throwing barrels at you.
(Philip J. Fry, *Futurama*)

You should really read this chapter if ...

- you don't really know what 'space' actually is.
- you have always wondered why you can't divide by bean curd.
- you don't believe that headbanging is encoded in the laws of nature.
- you don't know the link between coconuts and our planet's poles.

1.1 Space on stage

We all need some space. A big personal bubble around us. Far away places we can travel to, in order to develop our sense of time. Room, for improvement. Nooks and crannies, to cram away our junk. Territory, to fight over. The occasional piece of space in cake — or, if you are an astronaut celebrating your birthday at work, the other way round. Unknown lands, to explore. White space between letters, so we can take a breath and give meaning to words — why else did they make that bar the largest key on a laptop? Dark space to stare into on a starry summer night, so we can lie on our back and let the quiet void fire our philosophical thought generator.

'Why are we here?'

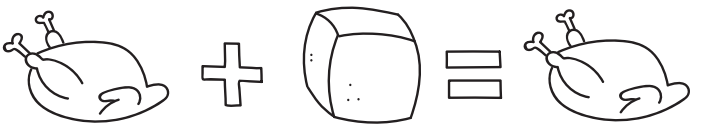
'Are we alone here, or is there something out there?'

'Does a block of tofu really exist or is it a soy-based illusion?'

Definition 1.

Tofu: *one of the biggest remaining mysteries in the field of Contemporary Culinary Philosophy, next to famous problems such as 'Is a cheese cake tart or pie?', 'Is it morally acceptable to put pineapple on a pizza?' and 'Is the hot dog a sandwich, or not?'*[7]

Not only philosophers, but also exact scientists are desperately trying to find out what tofu is. Based on the fact that nothing ever seems to change when you add it to a dish — tofu has neither taste, nor smell — mathematicians are claiming that tofu is the neutral element in the group of food additives. Put differently: it is the edible version of the number zero (you may remember from your maths classes that adding zero to something never changes anything, unless we are talking about school reports). Seeing tofu as a kind of zero also explains why you get total nonsense when you try to divide by it.



BASIC EXAMPLE OF TOFU ARITHMETIC

Theoretical physicists have taken a keen interest in tofu because it seems to be the only substance in the visible universe which does not contain traces of peanuts. This has led them to believe

7 America's National Hot Dog and Sausage Council (NHDSC) ruled that they are not. As the organisation's president put it: *Limiting the hot dog's significance by saying it is 'just a sandwich' is like calling the Dalai Lama 'just a guy'.*

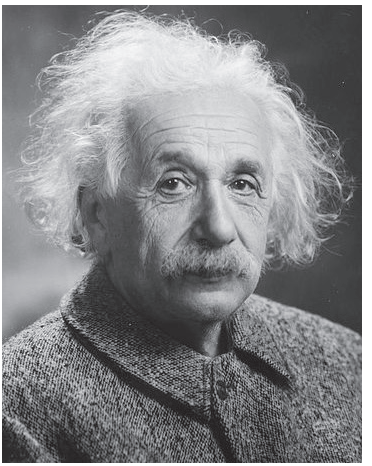
that bean curd is the absolute culinary vacuum: a zero-calorie state, which can only be obtained by extracting all the smell and flavour from matter. The only problem is that this does not explain the abundance of meat in the cosmos, which is why some people have conjectured the existence of dark tofu, the non-vegan counterpart of regular tofu. So far it has not yet been observed, but NASA (the National Aroma and Scent Administration) has plans to launch a space probe in 2022. This would be a natural follow-up to their most recent project: sending the Marslander Space-Eyed to our planetary neighbour, in order to see whether there once was whisky on the red rocks.

As to those other big questions of what space actually is, and where it came from, a mathematician already uses a completely different notion than, say, a philosopher or a physicist. For the latter, space is essentially a stage on which things exist, events happen and history unfolds. And just like love for furry purring kittens, it seems boundless, intangible and ubiquitous.[8] I often find myself wandering around this imaginary stage as if I were an extra in a movie without a script — utterly clueless and hoping for a free lunch — so this image does correspond with my gut feeling: space is a plane of existence, but then in three dimensions instead of two. However, mathematicians do not feel satisfied with this description. Like doing a job interview in a tracksuit: it may sound comfortable, but it is just not formal enough. What they want is a sound definition for space, an issue that I will try to address in a later chapter ('Setting Up Space').

Note that when physicists say three dimensions (backwards and forwards, right and left, up and down), they actually mean four. But this requires the inclusion of time as a fourth dimension — from past through present to future — which is precisely what Albert Einstein suggested when he stupefied the world with his theory of relativity, in the early 20th century. There are even people who claim that we live in a universe which has as many as

8 See also: the internet

ten or eleven dimensions — at least six of which are required in order to properly understand tofu, and one clogged up with lost socks. Some of these people spend their lives on a psychiatric ward wearing straitjackets, because they are said to be delusional. Others are even less fortunate and spend their lives as a string theorist.



Definition 2.

String theory: *an advanced mathematical framework in theoretical physics which attempts to address some of the most fundamental questions in black hole physics and early universe cosmology. It is seen as a serious candidate for the so-called Theory of Everything, a mathematical model describing all the fundamental forces of nature (gravity, electromagnetism, the weak nuclear force, the strong nuclear force and Jedi mind power) and all forms of matter. Until the day string theorists succeed in explaining why we sometimes have to rotate a USB-stick twice before it fits in the port, it can safely be ignored.*[9]

9 The mathematically schooled reader may see this as a proof that USB-sticks are spinors.

Definition 3.

Dimensions: *it is not easy to explain in full detail what a dimension exactly is, but luckily enough there is a nice parallel with the number of people having sex together. When there is just one person involved (one dimension), the only option is to move back and forth. Despite being a hands-on introduction to the world of dimensions, this is definitely not the most interesting case as it does not leave room for intersections. The next case (two dimensions) is definitely the most familiar one: it is taught at school and lends itself perfectly to visualisations on a screen, which may explain the many movies and websites covering this case. The situation in three dimensions is the most interesting one, as it allows manoeuvring around in a variety of ways which are inconceivable in the previous cases (left or right, in or out, top or bottom). Anything from four dimensions onwards makes most people feel rather uncomfortable, although some do prefer ten or even eleven (the swing theorists).*

Apart from the number of dimensions it has, the precise origin of space is also still a topic of heated debate amongst cosmologists and theoretical physicists. It actually divides them into competing camps quarrelling over, well, nothing really: one of the prevailing conjectures is that our universe — or even a multitude of parallel universes — came into existence out of absolute nothingness. Not unlike that adamant pimple that often sets up camp overnight, right in the middle of my forehead, where I somehow seem to lack the appropriate amount of skin to squeeze. There is probably even a parallel universe out there in which none other than Shakespeare fathered modern cosmology:

From zero to here? Oh!
A causative quantum coincidence.
Colliding clouds of quarks.
Coalescing chunks of matter.

Ecce,
the quirky creation of this celestial clusterfuck,
collectively referred to as our cosmos,
in a coconut shell.
To be from not to be,
that was the question.

(from 'Much Ado About Nothing')

All that is, was and will be, a universe much too big to see,[10] generated by some sort of abstract glitch: apart from a few religious fanatics — who seem to have taken this whole Big Bang Theory to a very unfortunate different level — this is of course where most of the religions and their followers pray to differ, as they all seem to have some sort of creation myth to adhere to. During my own struggle with a course on quantum field theory at university, I often asked myself: who needs physics when you can have something as simple as a cosmic egg, hatched by a celestial bird? Or an ancient Hindu story, according to which every life form on Earth originates from clarified butter — their take on *Ghee-nesis*, as worthy of spreading a Holy Word as can be.

Definition 4.

Quantum field theory: *this is another advanced framework in theoretical physics, aimed at explaining the behaviour of subatomic particles through a combination of special relativity and quantum mechanics. Despite being one of the most succesful frameworks ever devised by physicists, emerging from the work of generations of geniuses throughout the 20th century, quantum field theory itself lacks a rigorous mathematical foundation. So this means that from a mathematical point of view, quantum field theory is like holding*

10 Oh, did I already mention that you get extra points for spotting references to lyrics?